Parallel Computation for Natural Convection in Cavities

P.Wang and R.D.Ferraro

Jet l'repulsion laboratory
California, Institute of Technology

Abstract

Parallel computation for thermal convective flows in cavities with adiabatic horizontal boundaries and driven by differential heating of the two vertical end walls, is investigated using the Intel Paragon, Intel Touchstone Delta, and Cray T3D. A parallel computation code has been implemented by using a finite-diffcmlcc method with a multigrid elliptic solver and a Dufort-Frankel scheme. The domain decomposition techniques are discussed in detail. The parallel code is numerically stable, computationally efficient, and portable to various parallel architectures which support either PVM or NX libraries for communications. Finally, numerical results for various Rayleigh numbers and Prandtl numbers are presented.

1. INTRODUCTION

Convective motions driven by lateral temperature gradients in cavities are important in many areas of interest in industry and in nature. Applications include heating and ventilation control in building design and construction, cooling systems for nuclear reactors in the nuclear industry, solar-energy collectors in the power industry and some other areas. Due to the wide range of applications, studies of natural convection flow and heat transfer have been vigorously pursued for many years. A typical model of convection driven by a lateral thermal gradient consists of a two dimensional rectangular cavity with the two vertical end walls held at different constant temperatures. In order to determine the flow structure and heat transfer across cavities with different physical properties, numerous analytical, experimental and computational techniques have been used [1] [2] [4] [5] . The present study will focus on numerical simulation on large or small aspect ratios L cavity flows ($L >> 1$, or $L << 1$) with various Prandtl numbers and Rayleigh numbers by several parallel systems [3] [7] [8] [9].

2. MATHEMATICAL FORMULATION

The flow domain is a rectangular cavity of length l and height h. The appropriate governing equations, subject to the Boussinesq approximation, can be written in non-dimensional form as

$$\sigma^{-1}\left(\frac{\partial \omega}{\partial t} + J(\omega, \psi)\right) = \nabla^2 \omega + R\frac{\partial T}{\partial x}, \tag{1}$$

$$\nabla^2 \psi = -\omega, \tag{2}$$

$$\frac{\partial T}{\partial t} + J(T, \psi) = \nabla^2 T, \tag{3}$$

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$$\sigma = \frac{\nu}{\kappa}, \quad R = \frac{g\beta \, \Delta T h^3}{\kappa \nu} \tag{4}$$

are the Prandtl number and the Rayleigh number respectively. Here the pressure has been eliminated and

$$\omega = \frac{\partial w}{\partial x} - \frac{\partial u}{\partial z} \tag{5}$$

is the vorticity, with the two Jacobians given by

$$J(\omega, \psi) = \frac{\partial \omega}{\partial x}\frac{\partial \psi}{\partial z} - \frac{\partial \omega}{\partial z}\frac{\partial \psi}{\partial x}, \quad , \quad J(T, \psi) = \frac{\partial T}{\partial x}\frac{\partial \psi}{\partial z} - \frac{\partial T}{\partial z}\frac{\partial \psi}{\partial x}. \tag{6}$$

The boundary conditions on the cavity walls are

$$\psi = \frac{\partial \psi}{\partial x} = 0, \quad x = 0, L; \quad \psi = \frac{\partial \psi}{\partial z} = \frac{\partial T}{\partial z} = 0 \quad z = 0, 1 \tag{7}$$

$$T = 0, \quad x = 0; \quad T = 1, \quad x = L \tag{8}$$

for insulated walls, where $L = \frac{l}{h}$ is the aspect ratio of the cavity.

The equations possess an exact parallel-flow solution [3] [8] of a shallow cavity as:

$$T = x + c + R_1 F''(z), \quad \psi = R_1 F'(z) \tag{9}$$

where

$$F(z) = \frac{z^5}{120} - \frac{1}{48}z^4 + \frac{1}{72}z^3 - \frac{1}{1440} \tag{10}$$

$R_1 = \frac{R}{L^2}$, c is a constant which is determined by a full solution of the end-zone problems which will be discussed in the next paragraph For a tall cavity case, the parallel core structure is given as [1]:

$$T = T_c(x), \quad \psi = AF(x), \quad (0 < z < H), \tag{11}$$

where

$$T_c(x) = x, \quad F(x) = \frac{x^2}{24}(1 - X)^2. \tag{12}$$

and A is *the* Rayleigh number based on the cavity length.

For a shallow cavity, the following end-zone problem will be solved numerically *[3]:*

$$
\begin{array}{ccc}
 & \dfrac{\partial T}{\partial z} = 0 \quad (z = 1) & \\
\begin{array}{c} T = 0 \\ (x=0) \end{array} & \left|\begin{array}{l} \text{Semi-}\infty \text{ region} \\ \text{Boussinesq eqns. apply} \\ \text{Parameters } \sigma, R_1 = R/L \end{array}\right. & \begin{array}{l} T \sim x + c + R_1 F(z) \\ \psi \sim R_1 F'(z) \\ (x \to \infty) \end{array} \\
 & \dfrac{\partial T}{\partial z} = 0 \quad (z = 0) &
\end{array}
$$

and a end-zone problem of a tall cavity can be formulated by a similar approach.

3. NUMERICAL APPROACH AND PARALLEL COMPUTING TECHNIQUES

The finite difference method is used for the whole computation. DuFort-Frankel scheme is used for the vorticity (1) and the energy (3) equations, which is an explicit, three-layer method and has second-order accuracy. And the Multigrid method [6] is used for the Poisson equation (2). It has proven an effective and fast method. In present code, a complete V-Cycle scheme on four-level grids is used. The outer boundary conditions (9) and (11) were used for the end-zone problems of shallow and tall cavities respectively in the computation.

In the present study, the Intel Touchstone Delta, the Intel Paragon XP/S and the Cray T3D parallel systems were used for various computing experiments. In order to implement a parallel code with Dufort-Frankel-Multigrid method to natural convective flow problems in rectangular cavities, a two dimensional original fine mesh is partitioned into blocks of consecutive columns ($L >> 1$) or rows ($L << 1$) and distributed onto a linear array of processors (Figure 1 for a shallow cavity). This is a natural way for data partition with the above geometry as the communication among subdomains needs to be minimized.

P_1	P_2	......	P_{n-1}	P_n

Figure 1: Data partition on a shallow cavity with n processors

Communications will be encountered during the whole computation by using domain decomposition techniques. The major part of communications is that each subdomain needs to exchange information with its neighbors and this is done by direct message-passing NX or PVM software.

4. NUMERICAL RESULTS

Numerical results are obtained for various Rayleigh numbers and Prandtl numbers in a whole cavity or in semi-infinite regions of both tall arid shallow cavities. In Figure 2, a numerical solution for a large aspect ratio case is presented by the contour plots of stream function, vorticity, and temperature, which is in good argreement with the instability analysis. Here a mesh size of 16 X 2048 was USCd for the computation. In order to compare the performance 011 each system, 16 processors were used for the parallel code with the above numerical model. The computation results are showed in '1'able 1, which lists the total CPU time 011 the lest problem for the three systems. The Cray '1'311 gives the best performance, and the Paragon shows better performance than the Delta. By various tests on the parallel systems and comparisons with some previous results, the parallel code is proven numerically stable, efficient, and reliable.

Parameters	System	CPU Time(s)
$Mesh = 64 \times 1024$	Intel Touchstone Delta	1099
$R = 10^4, \sigma = 0.733$	Intel Paragon	824
$L = 16$	Cray T3D	197

'1'able 1: CPU times (seconds) using 16 nodes on the Delta, the Paragon, arid the Cray T3D for the problem with parameters noted in the '1'able.

The profiles in Figure 3 arc the scaling performance of parallel computation code for natural convective problems. Various meshes have been used with a test Ilo cIc1 of $R_1 = 400$, $\sigma = 0.733$, $L = 64$. The largest problem has a global grid of 256 x 32, 768 distributed 011 512 processors, which has a total unknowns 41,744,384. Figure 3(a) shows the ratio of execution time $T(n)/T(1)$ via the number of processors, and Figure 3(b) shows the scaling performance for large global grids 011 the Cray '1'31). These figures show that the speed up from 1 node to 128 nodes goes well, but starts to slow down when more processors come to play. It will be no longer the best strategy to partition the computational domain into blocks of columns if the number of processors is much larger than the aspect ratio L of a cavity. In this case, 2D partitioning should be applied, which will be considered in our future work.

5. CONCLUSIONS

The present end-zone problems contain only two parameters, R_1 and σ, instead of the three-parameter problem considered in numerical simulation of the full cavity flow [2] [5]. This approach can be used to provide approximations to the Nusselt number for all aspect ratios L provided L is sufficiently large for the conductive regime to apply in the core. This is a significant advantage of the asymptotic methods adopted here. And for a small aspect ratio, while the asymptotic structures are no longer valid, the code also works well for simulating a whole cavity flow. The speedup goes very well for cavity flows with a large aspect ratio, so the results can be used for comparing with some asymptotic theories based on large aspect ratios. The Cray T3D gives the best performance among the three machines. More work on much higher Rayleigh number problems will be considered on parallel systems, and work on 3D thermal convective flows is in the progress.

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References

[1] G. K. Batchelor. Heat transfer by free convection across a closed cavity between vertical boundaries at different temperatures. *Q. J. Appl. Math.*, 12:209 233, 1954.

[2] A. Bejan and C. L. Tien. Laminar natural convection heat transfer in a horizontal cavity with different end temperatures. *Trans. A.S.M.E.: J. Heat Trans*, 100:641-647, 1978.

[3] D. E. Cormack, L. G. Leal, and J. Imberger. Natural convection in a shallow cavity with differentially heated end walls. Part 1. Asymptotic theory. *J. Fluid Mech.*, 65:209 229, 1974.

[4] J. C. Patterson and S. W. Armfield. Transient features of natural convection in a cavity. *J. Fluid Mech.*, 219:469-497, 1990.

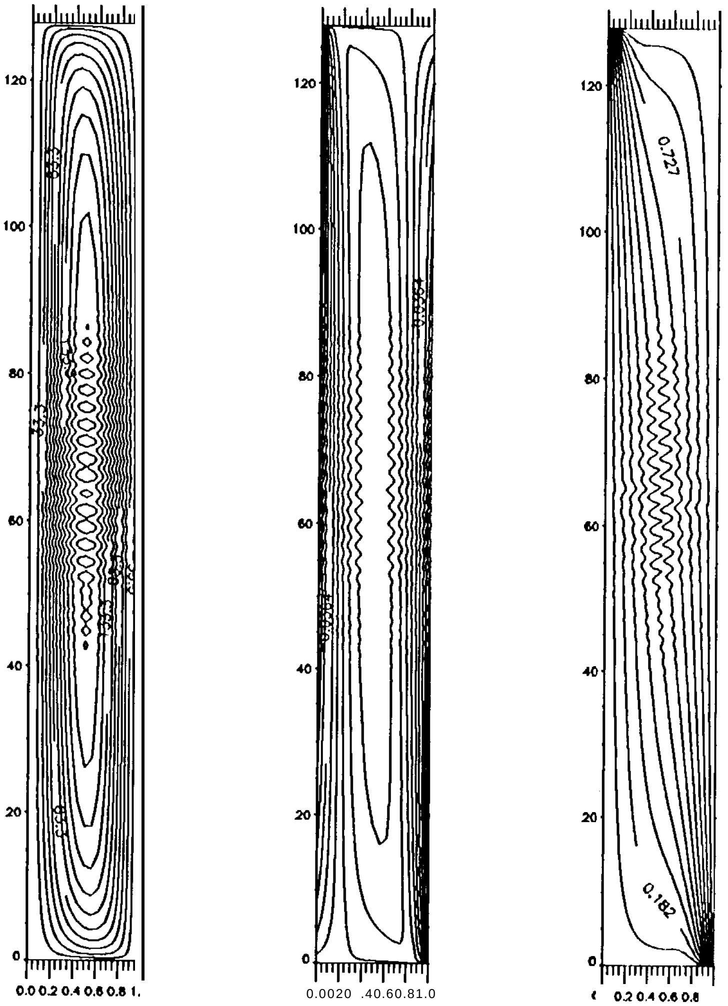
[5] J. E. Drummond and S. A. Korpela. Natural convection in a shallow cavity. *J. Fluid Mech.*, 182:543 564, 1987.

[6] A. Brandt. Multi-level adaptive solutions to boundary-value problems. *Math. Comp.*, 31:333 390, 1977.

[7] P. Wang and P. G. Daniels. Numerical solutions for the flow near the end of a shallow laterally heated cavity. *J. Eng. Math.*, 28:211-,226, 1994.

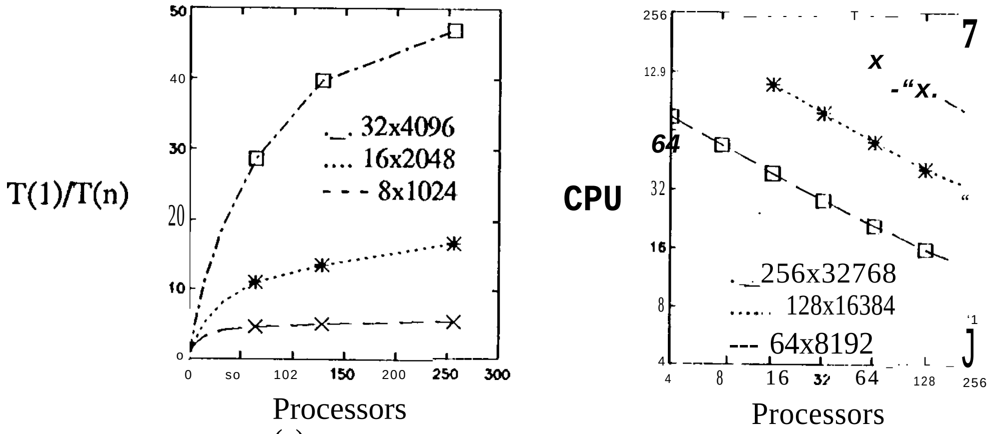
[8] P. G. Daniels, P. A. Blythe, and P. G. Simpkins. onset of multicellular convection in a shallow laterally heated cavity. *Proc. Roy. Soc. A*, 411:327-350, 1987.

[9] P. Wang and R. D. Ferraro. Parallel computation for natural convection. *Concurrency Practice and Experience, submitted*, 1995.



(a) (b) (c)

Figure 2: Contour plots for (a) stream function, (b) vorticity, (c) temperature for the insulating boundaries with Pr=6.983 , Ra=80,000 and L= 128.



(a) (b)

Figure 3: (a) Speed up of the parallel code on the Cray T3D. (b) Scaling performance on the Cray T3D.